A LIMIT ON NONLOCALITY FROM AN INFORMATION PROCESSING PRINCIPLE

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CONTEXT

Quantum mechanics Axioms

- Physical state : $|\psi\rangle \in \mathcal{H}$.
- Observable : $\mathcal{A}$ (hermitian operator).
- Measurement outcomes : a_n and $\mathcal{P}(a_n) = |\langle u_n|\psi\rangle|^2$.
- State after measurement : $\frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$
- Time evolution $i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$.

Special relativity Axioms

- The laws of physics are the same in all inertial frames of reference
- The speed of light in vacuum is the same in all inertial frames of reference

CONTEXT

Quantum mechanics Axioms

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Special relativity Axioms

Physical principles

↓

Mathematical representation

CONTEXT

Quantum mechanics Axioms	Special relativity Axioms
Mathematical representation ↓ Physical principles	Physical principles ↓ Mathematical representation

OUTLINE

1. Quantum nonlocality
2. Non-trivial communication complexity (NTCC)
3. Expanding the set of correlations that are known to violate NTCC

QUANTUM NONLOCALITY

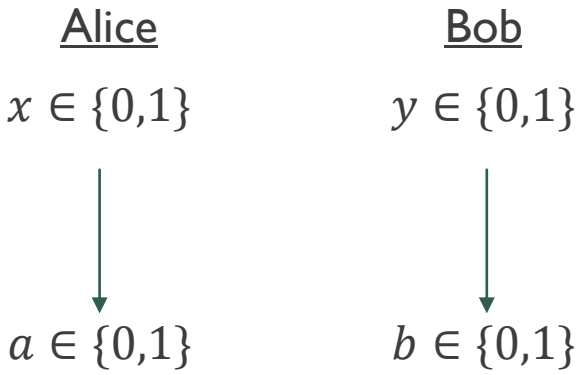
- Local theory (CHSH inequality) :

$$\mathcal{P}(a + b = xy) \leq 75\%$$

- Quantum (Tsirelson's bound) :

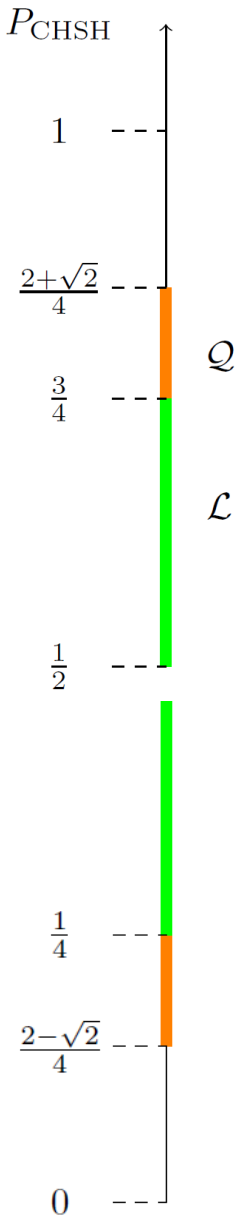
$$\mathcal{P}(a + b = xy) \leq \frac{2 + \sqrt{2}}{4} \approx 85.4\%$$

CHSH game

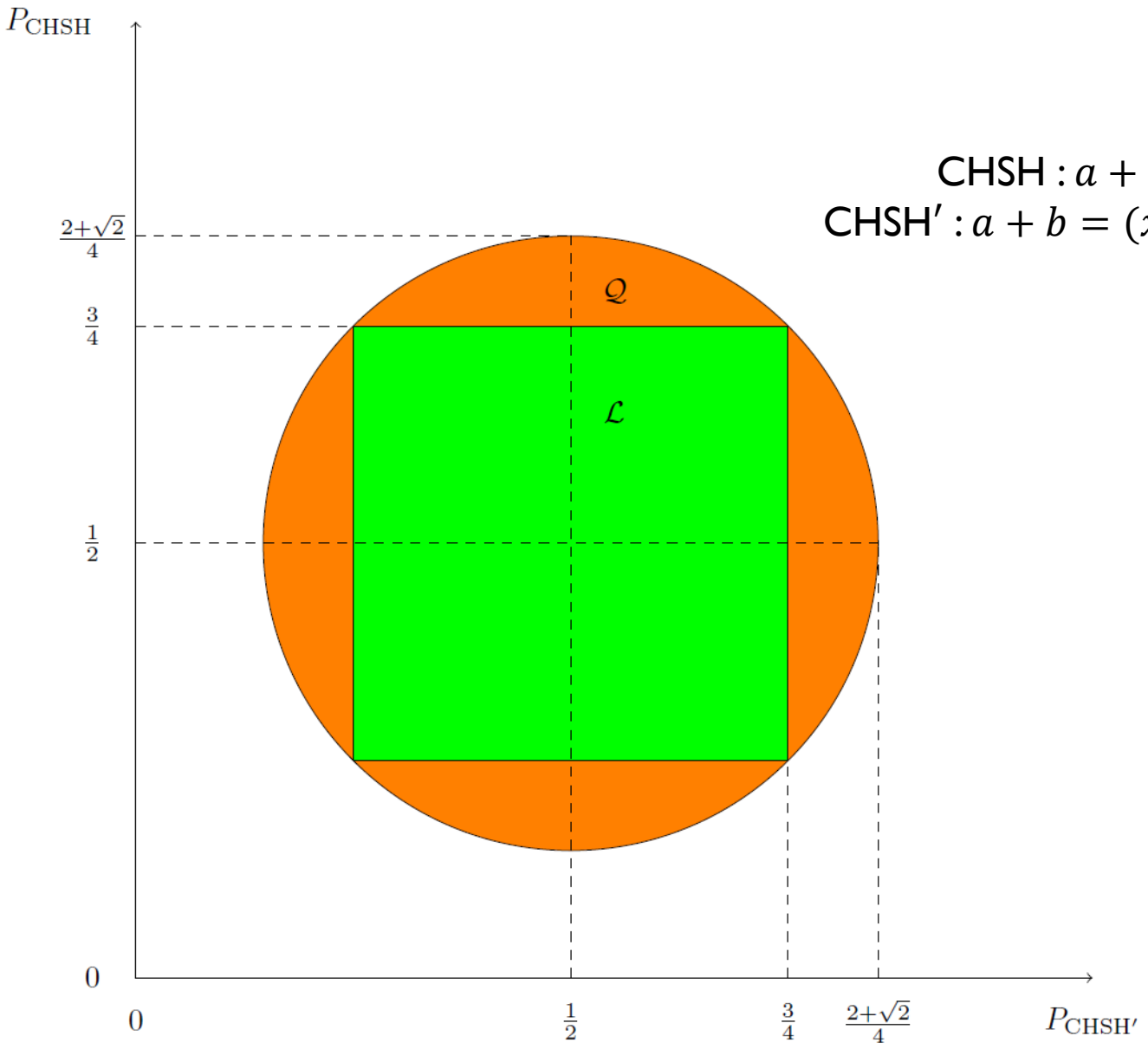


Condition : $a + b = xy$

Correlation : $\{\mathcal{P}(a, b|x, y)\}$


P_{CHSH}
1
$\frac{2+\sqrt{2}}{4}$
$\frac{3}{4}$
$\frac{1}{2}$
$\frac{1}{4}$
$\frac{2-\sqrt{2}}{4}$
0
$\mathcal{Q}$
$\mathcal{L}$

CHSH : $a + b = xy$



CHSH : $a + b = xy$

CHSH′ : $a + b = (x + 1)(y + 1)$

PHYSICAL PRINCIPLE

Candidate :

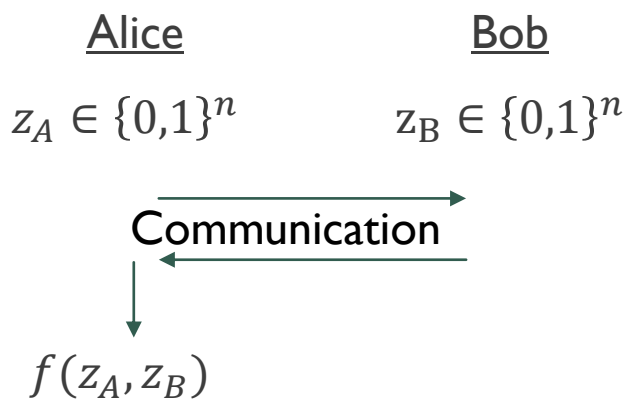
Non-trivial communication complexity

Non-trivial communication complexity (NTCC)

Physical principle according to which communication complexity should not be trivial.

COMMUNICATION COMPLEXITY (CC)

$f: \{0,1\}^n \times \{0,1\}^n \rightarrow \{0,1\}$

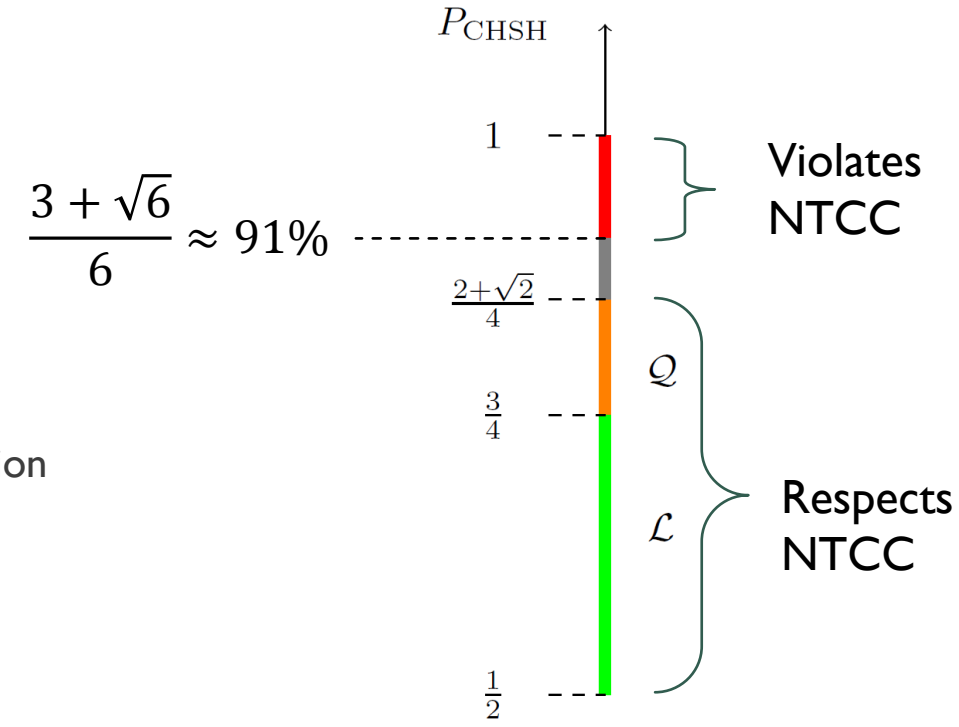


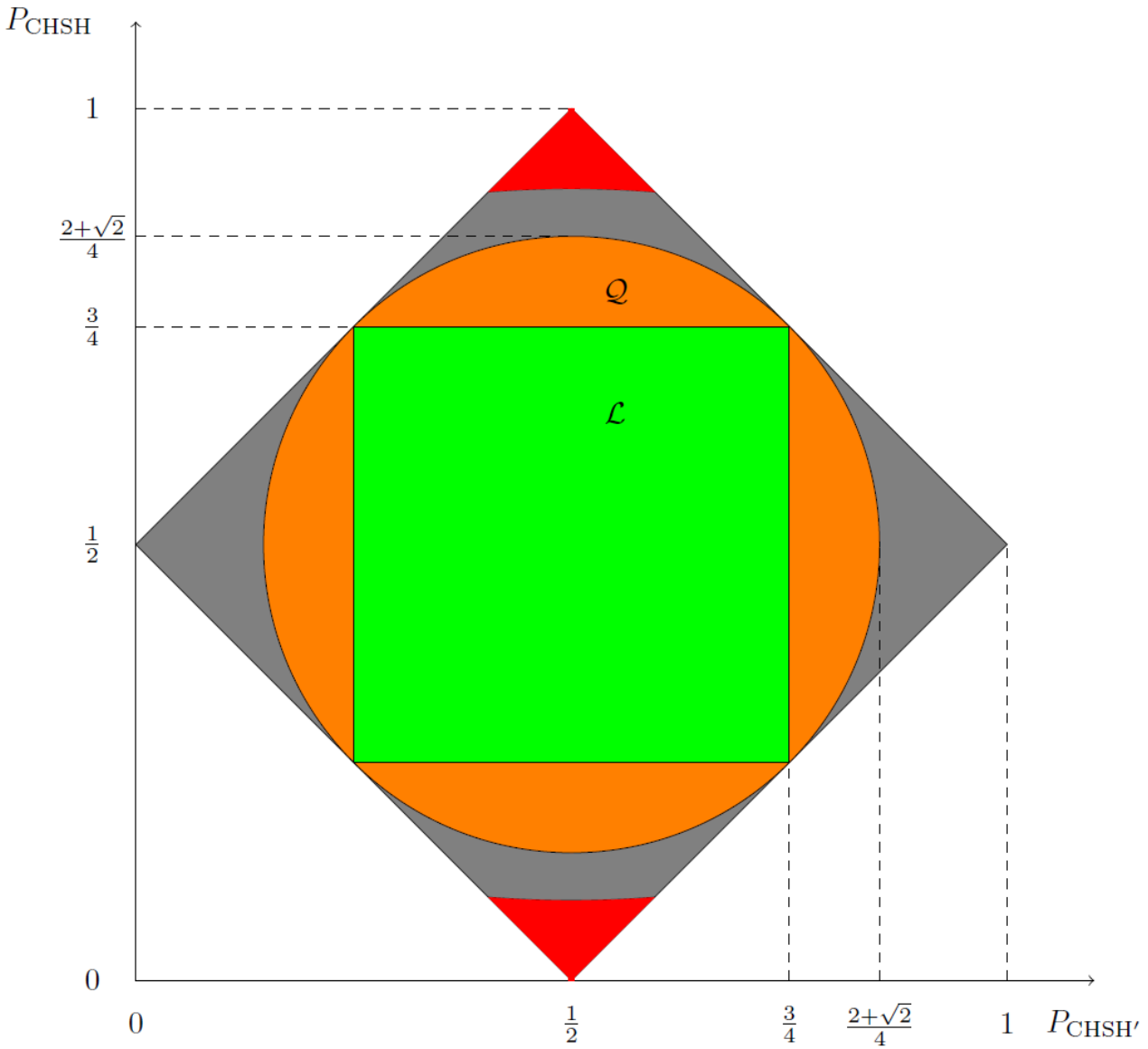
$CC(f)$: Minimum number of bits of communication for Alice to learn $f(z_A, z_B)$.

Trivial CC : In a physical theory, CC is trivial if $\forall f, CC(f) \leq c$ with c a constant.

NON-TRIVIAL COMMUNICATION COMPLEXITY

- [BBLMTU'06]
  - Protocol for Alice and Bob
  - Exploits nonlocal correlations
  - Can be used to compute any function with one bit of communication (trivial CC)

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$\frac{3}{4}$
$\frac{1}{2}$
0
$\mathcal{Q}$
$\mathcal{L}$
0
$\frac{1}{2}$
$\frac{3}{4}$
$\frac{2+\sqrt{2}}{4}$
1
$P_{\mathrm{CHSH}'}$

OPEN QUESTIONS

- Gap between quantum correlations and NTCC violation
- Other correlations
- Multipartite correlations?