Systems illuminated by squeezed wave-packet modes

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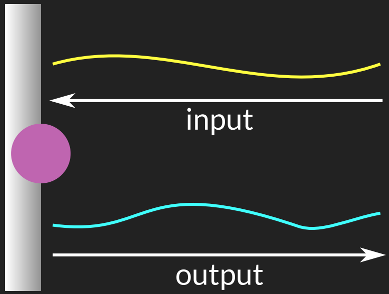
Open systems are subject to quantum noise



Gaussian white noise is convenient

$$
\begin{aligned}
[b_s, b_t^\dagger] &= \delta(s-t) \\
\langle b_t \rangle &= \beta_t \\
\langle b_s^\dagger b_t \rangle &= N\delta(s-t) \\
\langle b_s b_t \rangle &= M\delta(s-t)
\end{aligned}
$$

White because of δ functions

Gaussian because only first and second moments matter

Gaussian white noise is convenient

$$\dot{\rho}_t = \sqrt{\gamma}[\beta^* L - \beta L^\dagger, \rho_t] \\ + \gamma \left[(N+1)\mathcal{D}[L]\rho_t + N\mathcal{D}[L^\dagger]\rho_t \\ + \tfrac{1}{2} M^* [L, [L, \rho]] + \tfrac{1}{2} M [L^\dagger, [L^\dagger, \rho]] \right]$$

System dynamics described by simple Markovian master equation

White noise isn't physical

$$
\begin{aligned}
[b_s, b_t^\dagger] &= \delta(s-t) \\
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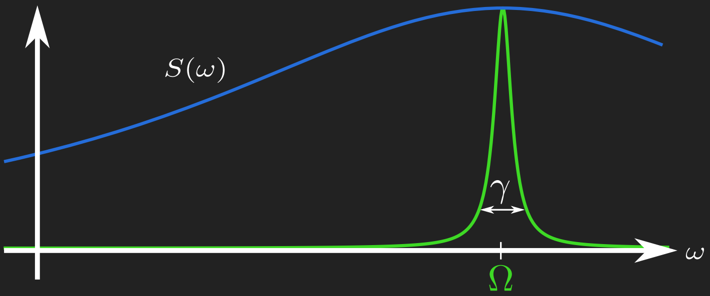
\end{aligned}
$$

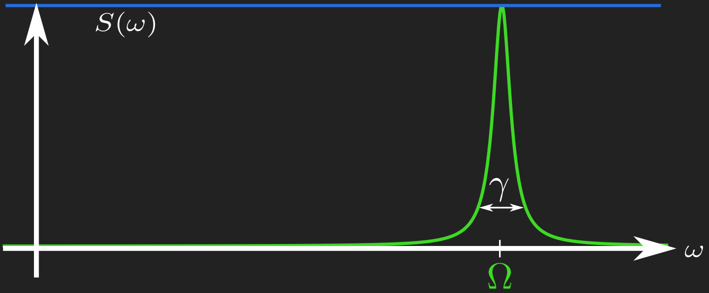
δ correlations in the time domain imply constant power spectral density.

White noise is often a good approximation



System can't tell the difference if noise bandwidth is broad compared to system linewidth.

White noise is often a good approximation



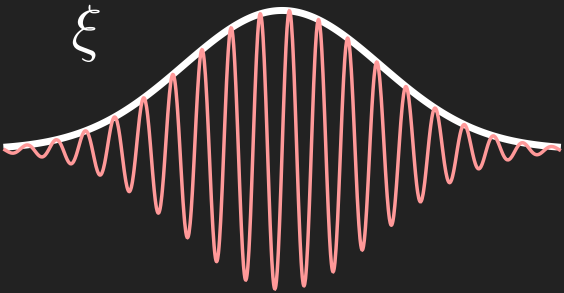
System can't tell the difference if noise bandwidth is broad compared to system linewidth.

Sometimes white noise is a bad approximation

If we try to describe a system subjected to squeezed noise where we perform photodetection on the output field, approximating both the input noise and the photodetector as broadband leads to an explosion!

A strategy for avoiding this problem is to restrict non vacuum statistics to discrete wave packets.

Wave packets are field modes



$$B[\xi] = \int dt\, \xi_t^* b_t$$

$$[B[\xi], B^\dagger[\xi]] = 1$$

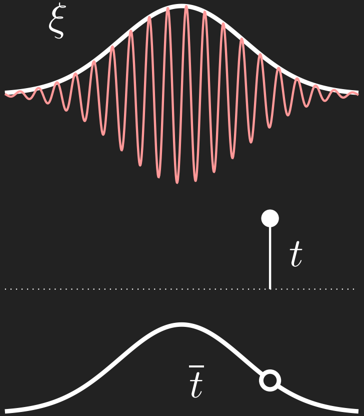
Wave-packet coherent states are simple

$$\begin{aligned}|\alpha[\xi]\rangle &= \mathsf{exp}\left(\alpha B^\dagger[\xi] - \alpha^* B[\xi]\right)|\mathsf{vac}\rangle \\ &= \mathsf{exp}\left[\int dt\left(\alpha\xi_t b_t^\dagger - \alpha^*\xi_t^* b_t\right)\right]|\mathsf{vac}\rangle\end{aligned}$$

The exponent commutes at different times, so the field is in a temporal product state and we still have a Markovian master equation with $\beta_t = \alpha\xi_t$:

$$\dot{\rho}_t = \sqrt{\gamma}[\beta_t^* L - \beta_t L^\dagger, \rho_t] + \gamma\mathcal{D}[L]\rho_t$$

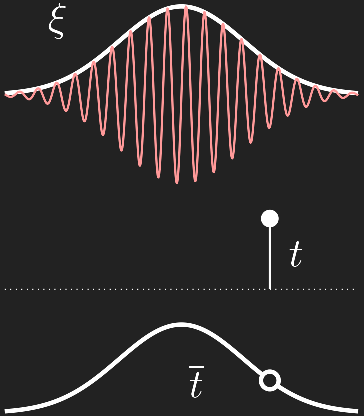
Wave-packet number states are trickier



$$|\psi_\xi\rangle = |0_t\rangle \otimes |\tilde{\psi}^0_{\bar{t}}\rangle + |1_t\rangle \otimes |\tilde{\psi}^1_{\bar{t}}\rangle$$

$$|n[\xi]\rangle = \frac{B^\dagger[\xi]^n}{\sqrt{n!}}|\mathsf{vac}\rangle$$

Wave-packet number states are trickier



$$|\psi_\xi\rangle = |0_t\rangle \otimes |\tilde{\psi}^0_{\bar{t}}\rangle + |1_t\rangle \otimes |\tilde{\psi}^1_{\bar{t}}\rangle$$

$$|n[\xi]\rangle \sim |0_t\rangle \otimes |n_{\bar{t}}\rangle + |1_t\rangle \otimes |(n-1)_{\bar{t}}\rangle$$

Can't write a Markovian master equation

$$\rho_t = \mathsf{tr}\left(U_{t,0}\,\rho_0 \otimes |n[\xi]\rangle\langle n[\xi]|\,U^\dagger_{t,0}\right)$$

Want:

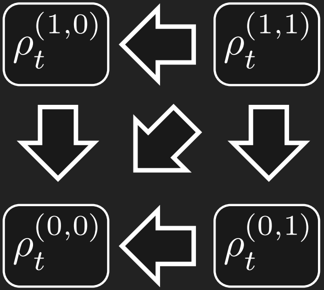
$$\dot{\rho}_t = f(\rho_t)$$

Can't write a Markovian master equation

$$\begin{aligned}\dot{\rho}_t &= \dot{\rho}_t^{(n,n)} \\ &= f(\rho_t^{(n,n)}, \rho_t^{(n-1,n)}, \rho_t^{(n,n-1)}, \rho_t^{(n-1,n-1)}) \\ \rho_t^{(m,n)} &= \mathsf{tr}\left(U_{t,0}\,\rho_0 \otimes |m[\xi]\rangle\langle n[\xi]|\,U_{t,0}^{\dagger}\right)\end{aligned}$$

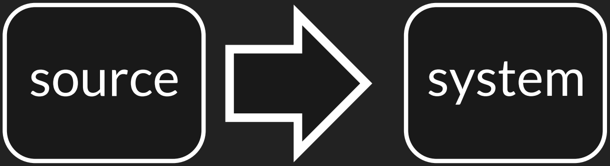
N-photon wave packets interacting with an arbitrary quantum system, B.Q. Baragiola, R.L. Cook, A.M. Brańczyk, and J. Combes, Phys. Rev. A **86**, 013811 (2012).

Have a hierarchy of equations



$$\rho_0^{(m,n)} = \delta_{mn}\rho_0 \sim \mathbb{1} \otimes \rho_0$$

Similar to cascaded systems



$$\rho_0 = \rho_{\text{source},0} \otimes \rho_{\text{system},0}$$

n-dimensional source can be mocked up to produce the n-photon wave packet.

Calculate different “extra” information

A solution to the source model master equation gives all expectation values of the source system.

A solution to the hierarchy gives system solutions for all possible wave-packet initial states.

Hierarchy gives solutions for many states

$$\rho_{\text{wavepacket},0} = \sum_{m,n} \varrho_{mn} |m[\xi]\rangle\langle n[\xi]|$$

$$\rho_t = \mathrm{tr}\left(U_{t,0}\,\rho_0 \otimes \rho_{\text{wavepacket},0}\,U_{t,0}^{\dagger}\right)$$

$$= \sum_{m,n} \varrho_{mn}\rho_t^{(m,n)}$$

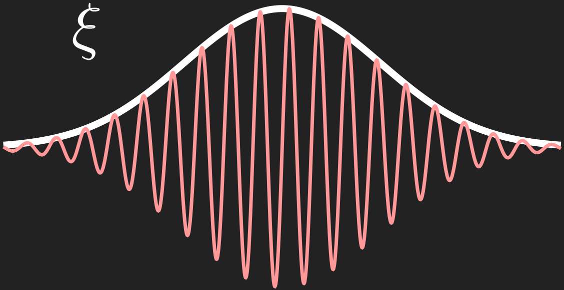
Can approximate squeezed states

Truncate the squeezed state at some maximum photon number n.

Seems inefficient, since squeezed vacuum only has support on even-photon-number states.

Try a more tailored approach.

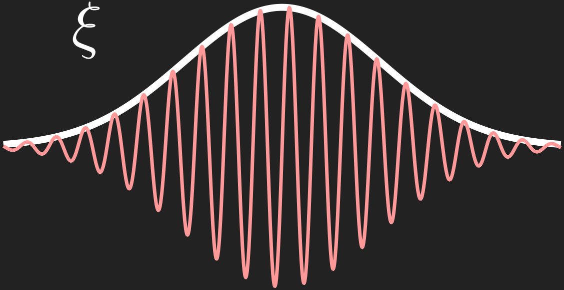
Squeezed wavepackets



$$S_\gamma[\xi] = \mathsf{exp}\left[\tfrac{1}{2}\left(\gamma^* B[\xi]^2 - \gamma B^\dagger[\xi]^2\right)\right]$$

$$B[\xi] = \int dt\, \xi_t^* b_t$$

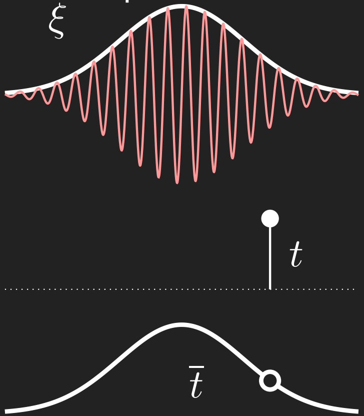
Squeezed wavepackets have temporal correlations



$$\langle b_t^\dagger b_s \rangle = \xi_t \xi_s^* N$$

$$\langle b_t b_s \rangle = \xi_t^* \xi_s^* M$$

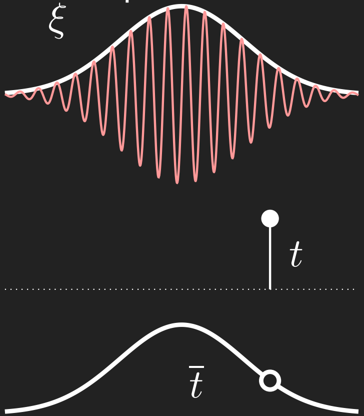
Squeezed decomposition



$$|\psi_\xi\rangle = |0_t\rangle \otimes |\tilde{\psi}^0_{\bar{t}}\rangle + |1_t\rangle \otimes |\tilde{\psi}^1_{\bar{t}}\rangle$$

$$S_\gamma[\xi] = e^X S_\gamma[\bar{t}] e^{-X}$$

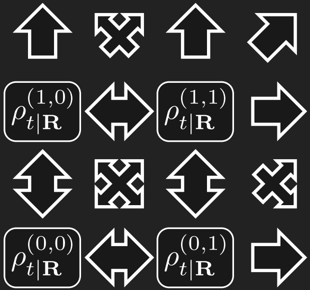
Squeezed decomposition



$$|\psi_\xi\rangle = |0_t\rangle \otimes |\tilde{\psi}^0_{\overline{t}}\rangle + |1_t\rangle \otimes |\tilde{\psi}^1_{\overline{t}}\rangle$$

$$\big|n_\gamma[\xi]\big\rangle \sim |0_t\rangle \otimes \big|n_{\gamma,\overline{t}}\big\rangle + |1_t\rangle \otimes \Big(\big|(n-1)_{\gamma,\overline{t}}\big\rangle + \big|(n+1)_{\gamma,\overline{t}}\big\rangle\Big)$$

Squeezed couples ←, →, ↓, ↑!



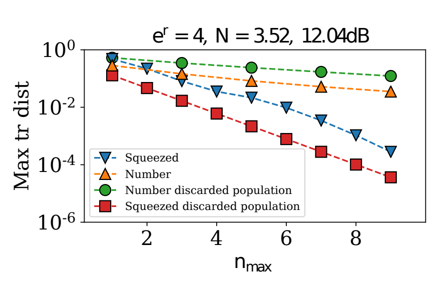
$$\dot{\rho}_t^{(m,n)} = f\left(\rho_t^{(m,n)}, \rho_t^{(m-1,n-1)}, \ldots, \rho_t^{(m+1,n+1)}\right)$$

Difficult to interpret

Truncating in the number basis gives a sequence of better approximations to a squeezed wave packet. Each approximation is physical.

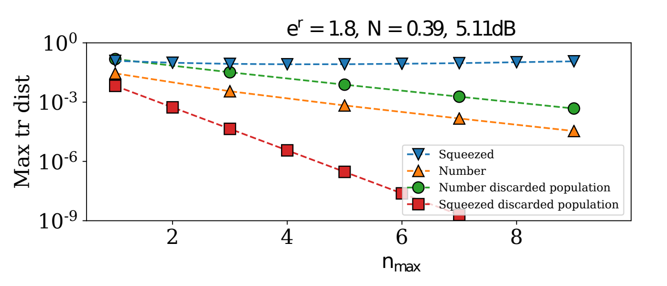
Truncating the squeezed hierarchy ought to give a sequence of better approximations as well, but these approximations are not guaranteed to be physical.

Numerical evidence



For the master equations, the squeezed hierarchy converges much more quickly that the number hierarchy does.

Numerical evidence



Stochastic photon-counting master equations behave poorly in the squeezed hierarchy.

Summary

1. Equation hierarchies provide a nice framework for non Markovian evolution
2. For master equation evolution, tailoring the formalism for squeezing boosts performance
3. The unbounded nature of the squeezed hierarchy appears to cause problems for conditional evolution