Fighting Kerr-Induced Non-Gaussianity with Squeezing

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Goal

Create and preserve superpositions of large classical states—Cat states

\[ \frac{1}{\sqrt{2}} |\text{Cat}\rangle + \frac{1}{\sqrt{2}} |\text{Mouse}\rangle \]

Classical state \(\rightarrow\) coherent state in a lossy cavity
Coherent states

- Coherent state:
  \[ |\alpha\rangle \quad \text{(Cat alive)} \]
  \[ |\alpha e^{i\pi}\rangle \equiv | -\alpha\rangle \quad \text{(Cat dead)} \]

- Poissonian photon number distribution:
  \[ |\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \]

- Quasiprobability distribution, the Wigner function:
  \[ W(\beta, \beta^*) = \int e^{\lambda^* \beta - \lambda \beta^*} \text{Tr}[\rho e^{\lambda a^\dagger - \lambda^* a}] d^2\lambda \]
  \[ W_{\text{coh}, \pm \alpha}(\beta, \beta^*) = \frac{2}{\pi} e^{-2|\beta \mp \alpha|^2} \]
  Gaussian of width \( 1/\sqrt{2} \)
Squeezed states

\[ \Delta X \Delta Y = \frac{1}{2} \]

\[ \Delta X = \frac{1}{\sqrt{2}} e^{-r}, \Delta Y = \frac{1}{\sqrt{2}} e^{r} \]

\[ \Delta X = \frac{1}{\sqrt{2}} e^{r}, \Delta Y = \frac{1}{\sqrt{2}} e^{-r} \]

\[ \Delta X' = \frac{1}{\sqrt{2}} e^{-r}, \Delta Y' = \frac{1}{\sqrt{2}} e^{r} \]
Cat States

- Photon number distribution in a Cat state:

\[
|\psi_\pm\rangle = |\alpha_0\rangle \pm |-\alpha_0\rangle
\]

\[
|\alpha_0\rangle = e^{-\frac{1}{2}|\alpha_0|^2} \sum_{n=0}^{\infty} \frac{(\alpha_0)^n}{\sqrt{n!}} |n\rangle
\]

\[
|-\alpha_0\rangle = e^{-\frac{1}{2}|\alpha_0|^2} \sum_{n=0}^{\infty} \frac{(-\alpha_0)^n}{\sqrt{n!}} |n\rangle
\]

\[
|\psi_+\rangle \quad \text{Even photon number}
\]

\[
|\psi_-\rangle \quad \text{Odd photon number}
\]
Cat States

- Wigner function of Cat states:

\[ |2\rangle + |2\rangle \]

\[ |2\rangle - |2\rangle \]
• Cat states are the essence of quantum computing with Cat codes.

• Qubit based quantum computing requires a large number of qubits for quantum error correction.

• The large Hilbert space of a cavity enables redundant encoding of information for QEC.

• Source of error -> single photon loss from the cavity.

• Only one error syndrome -> photon number parity measurement.

\[
|0\rangle \equiv |\alpha\rangle + | - \alpha\rangle \quad \xrightarrow{\text{Single photon loss}} \quad |\alpha\rangle - | - \alpha\rangle \\
|1\rangle \equiv |i\alpha\rangle + | - i\alpha\rangle \quad \xrightarrow{\text{ }} \quad |i\alpha\rangle - | - i\alpha\rangle
\]

Even photon number \quad Odd photon number

Z. Leghtas et al. PRL 111 120501 (2013)
M. Mirrahimi et al. NJP 16 045014 (2015)
• A cavity cat state can be created by using its dispersive interaction with a qubit:

\[ \hat{H} = \chi a^\dagger a \sigma_z \]

Generating Cat States

![Diagram showing spectroscopy frequency and normalized spectroscopy signal]

\[ |\beta\rangle, |\beta\rangle + |\beta\rangle, |\beta\rangle - |\beta\rangle \]

\[ \beta \sim 2.3 \]

Effect of Kerr term on generation of Cat states

- Higher order Kerr term in the dispersive approximation:
  \[ \hat{H} = \chi a^\dagger a \sigma_z - K a^\dagger a^2 \sigma_z + ... \]

- Gate time increases: \[ \tau = \frac{\pi}{2(\chi + K - K\langle a^\dagger a \rangle)} \]

- The final cat state is distorted i.e., it's no longer a superposition of two coherent states.

\[ |5\rangle + |-5\rangle, \ d=100 \text{ photons} \]
Effect of Kerr term on generation of Cat states

$|7\rangle + | -7\rangle$

$\text{d}=196 \text{ photons}$

$|10\rangle + | -10\rangle$

$\text{d}=400 \text{ photons}$
Effect of Kerr term once a Cat state is created

- Once a cat state is created, its shape can be distorted by the Kerr term

\[ \hat{H} = K a^\dagger a^2 \]
Revisiting Kerr-nonlinearity

- Driven Harmonic oscillator
  \[ \hat{H} = \delta a^\dagger a + \mathcal{E}(a^\dagger + a) \]

\[ t = 0 \]

- Bistability in driven anharmonic oscillator
  \[ \hat{H} = \delta a^\dagger a - K a^\dagger 2 a^2 + \mathcal{E}(a^\dagger + a) \]

\[ t = 0 \]
The probability of occupation of these states depends on the quantum fluctuations.

Thus, by squeezing the fluctuations in the “correct” quadrature one can force the system to be in one of the two states.

\[ \hat{H} = \delta a^\dagger a - K a^\dagger a^2 + \mathcal{E}(a^\dagger + a) + \mathcal{E}_s (a^\dagger e^{i\phi} + a e^{-i\phi}) \]

\( \mathcal{E}_s = 0 \)
\( \mathcal{E}_s \neq 0, \quad \phi = 0 \)
\( \mathcal{E}_s \neq 0, \quad \phi = \pi \)

C. M. Savage and D. F. Walls PRL 57 2164 (1987)
Cat states with Squeezing

Ideal dispersive cavity-qubit Hamiltonian
\[ \hat{H} = \chi a \dagger a \sigma_z \]

Cavity-qubit Jaynes-Cummings Hamiltonian without dispersive approx.

Cavity-qubit Jaynes-Cummings Hamiltonian without dispersive approx. but with squeezing
Cat states with Squeezing

- Ideal dispersive cavity-qubit Hamiltonian
- Cavity-qubit Hamiltonian without dispersive approx.
- Cavity-qubit Hamiltonian without dispersive approx. and squeezing
The fidelity of the corrected state depends only on the cavity loss, unlike correction schemes with a qubit.

Conclusion and future directions

- Possible to counteract the effect of Kerr nonlinearity with squeezing.
- Cat states with up to 400 photons can be prepared with 98% fidelity with ~ 4 dB squeezing.
- Effective stabilization of cat states.
- Current simulation results are for a parametrically driven cavity i.e., the squeezing is produced inside the cavity.
- More simulations with the source of squeezing moved outside the cavity.

\[ \frac{1}{\sqrt{2}} |\text{cat}\rangle + \frac{1}{\sqrt{2}} |\text{anti-cat}\rangle \]